The Study on Patent Acquisition from Complementarity and Supplementarity: Evidence from Smartphones of Apple and Samsung

Ping-Chun Chang[1], Yu-Hsin Chang[1], Fang-Pei Su[2], Shu-Jung Chen[1], Kuei Kuei Lai[1]
[1]National Yunlin University of Science and Technology, Dept. of Business Administration, Douliou, Yunlin, Taiwan, ROC
[2]Shu-Te University, Dept. of Logistics Management, Yanchao, Kaohsiung, Taiwan, ROC

***Abstract*--This study tries to figure out the strategy of patent acquisition against infringe litigation between rival companies of smartphone under dynamic competition. Patent citation network analysis is used to realize the difference of patent deployment and portfolios after patent acquisition between the leader Apple and the follower Samsung from the view of supplementary and complementary. Four patent indexes provide the movement of technology position and role in the network and the change in technology supplementary/complementary. The result shows that even though the leader and follower used to cooperate in relationship of OEM with technology supplementary, they intend to "de-opponent" technically because of patent litigation after becoming rivals in the same market. The leader will acquire supplementary patent to enhance his original patent portfolios and the follower will go the other way to strengthen his patent portfolios by acquiring complementary patent.**

I. INTRODUCTION

The demand of mobile internet device promotes severe competition in smartphone market and triggers global patent war among the smartphone companies[1]. How to win from the competition of patent deployment? How to response to opponents in the dynamic competition [2-4]? How to protect the company by patents? These factors are crucial for the change of market position.

Recently, Apple and Samsung have severe patent litigation against each other. Samsung has followed the leader, Apple, and been a free rider for a couple of years [5]. Samsung used to be the long-term OEM partner of the supply chain for Apple but now Samsung wants to be independent for its brand. In spring 2011, Apple fired the first shut of patent litigation to Samsung and meanwhile brought Motorola Mobility to the frontline. In August 2011, Apple and Samsung had nineteen gun fights of patent litigation in nine countries. These two companies shared more than half market of smartphone and had a war of fifty patent litigations globally.

These two companies speed up patent battles and patent acquisition in dynamic coopetition because of the similarity of market and resource. Patent litigation could deliver message to the market and interfere with opponent's new product. Patent acquisitions could strengthen patent portfolios to attack or protect from opponent. Thus, how do the leader, Apple, and the follower, Samsung, gain the advantage in dynamic competition by patent acquisition? How to choose patent candidates for acquisition to strengthen patent portfolios? How to detect the strategic intention of opponents for patent acquisition? These issues are very important for patent deployment of smartphone companies.

II. PATENT CITATION NETWORK ANALYSIS

Technological network is similar to social network [6, 7]. Technological network could be represented by patent citation network which treats patents as notes and citations as relation ties [7-11]. Social network is a set of actors and relational ties[12, 13]. The actors in a network could be persons, organizations or technologies. The relation based on the research of patent network can be citation if the patents act like actors[8, 9, 14]. Patent citing prior art forms the relation between both patents. Fig. 1 is a simple patent citation network whose nodes are patents. The relational ties show that the citing patents cite the cited patent.

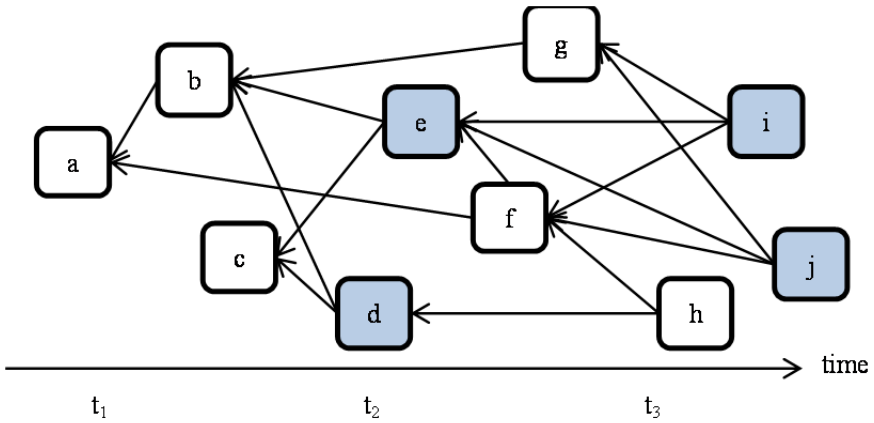


Fig. 1: Patent Citation Network

One kind of network is the whole network with a certain boundary and the other one is the ego-network which expands from an ego center without any preset boundary[13, 15, 16]. Chen and Lai call this ego network as an inside-out analysis method to search and identify the cooperative partner from its core technologies and its own view. This research deploys this inside-out approach, ego network, to analyze patent candidates for acquisition.

III. SUPPLEMENTARITY AND COMPLEMENTARITY FOR PATENT ACQUISITION

The relation of cooperation within the industrial organization consists of two different concepts, supplementary and complementary[5, 17]. The cooperation includes several functions such as knowledge, technology and product[11, 18, 19]. Based on this view, this study discusses the strategy of patent acquisition between leading and following companies by the concept of supplementary and complementary of patent and technology[18, 20].

Four indexes of TKS, TKR, CIK and CEK are used for this study[21]. Technological Knowledge Status, TKS, and Technological Knowledge Reliability, TKR, could figure out the position and role of companies and the movement of position within the network because of acquisition. Common Internal Knowledge, CIK and Common External Knowledge, CEK could measure the supplementary and complementary of patent and technology. Patent citation represents not only the direct dependency for technology but also common knowledge between patents [8]. A patent citing a prior patent shares the supplementary knowledge with the prior art[22]. In the other word, the more two companies' patents cite each other, the more common knowledge they share. Chen and Lai (2012) call this overlap of common knowledge as 'common internal knowledge', CIK, to measure the supplementary knowledge between two companies. Secondly, the dual indirect ties from the third actor could create innovative activity for the couple of actors[23]. Therefore, these indirect ties from the third actor shows that this couple of actors have more common knowledge confirmed by outsider but less similarity of knowledge shared by each other. Chen and Lai [24] call this overlap from outsiders as 'Common External Knowledge', CEK, to measure the complementary knowledge between two companies.

This research, first, reviews the background of patent litigation and patent acquisition of the leader and follower in smartphone industry, then explores the movement of role and position of companies and measures the change of supplementary/complementary of technology by the patent citation network, finally, addresses the conclusion and suggestion.

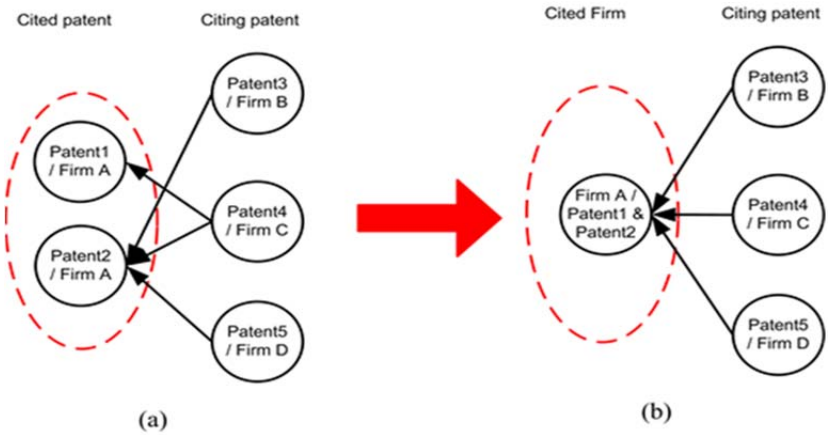


Fig 2: Concept of Technological Knowledge Status, TKS
Source: Chen & Lai(2012)

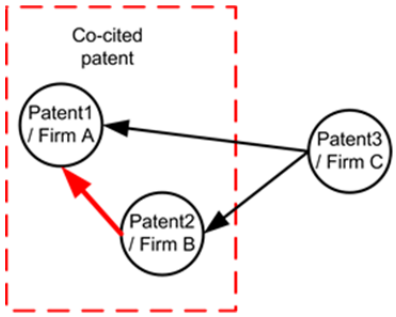


Fig 3: Concept of Technological Knowledge Reliability, TKR
Source: Chen & Lai(2012)

IV. DATA AND ANALYSIS METHOD

Patents of litigation between two smartphone manufactures, Apple and Samsung, are collected from the LexisNexis database from 2011 to 2012. Forty two patents in total with litigation are found from April 15, 2011 to April 18, 2012, as Fig 4[25]. Then, based on these 42 patents, we retrieve 1,422 patents relating to smartphone citing or cited by these 42 patents. These 1,422 patents relate to the 42 ego center in the inside-out ego network.

We cut 1,422 patents down to 459 ones by taking out the patents cited counts below 4. The row matrix is the citing patents and the column matrix is the cited patents in the patent citation matrix $[\varepsilon_{ij}]_{459\times459}$. Due to co-cited patents being the important part of analysis procedure, we take out 289 patents with co-cited counts below 2 and 3 isolated patents from column matrix and 236 patents with no citing counts from the row matrix leaving the modified matrix $[\varepsilon_{ij}]_{223\times167}$.

At first, citing and cited patents construct the patent citation network. Then, the cited patents belonging to one company are combined together to be the set of company of column matrixes. This new matrix is a 2-mode affiliate network formed by the patents affiliating to companies. Transform the matrix $[\varepsilon_{ij}]_{223\times167}$ into the affiliate matrix $[\alpha_{kr}]_{223\times93}$. Then, we calculate TKS, TKR, CIK and CEK from this matrix $[\alpha_{kr}]_{223\times93}$ as patent indexes to further analysis.

V. RESULTS

A. Technological Knowledge Status, TKS, and Technological Knowledge Reliability, TKR

TKS of IBM、Apple、AT&T、Synaptics and Xerox are the highest five, and Samsung much falls behind with TKS before patent transference in Fig 5. However, the positions of technology of these six company move obviously in the TkS-TKR diagram. Product property and evaluation index before/ after patent transference of the six main companies show in table 1. Patent counts of these main companies before patent transference are Apple(22)、IBM(13)、Xerox(8)、Synaptics(7)、AT&T(5). Patent counts after patent transference are Apple(24)、IBM(8)、Xerox(8)、Synaptics(7)、Samsung(4)、AT&T(4). Only Appleand Samsung increase their patent counts, the patent counts of IBM and AT&T decrease, Xerox and Synaptics hold constant with patent counts. Fig. 6 shows that Apple keeps the advantage of technolodgy and produces patent continuously. IBM had no patent output after patent transference. The patent counts of Samsung has increased a lot since 2009 after transference. The others hold steady after transference.

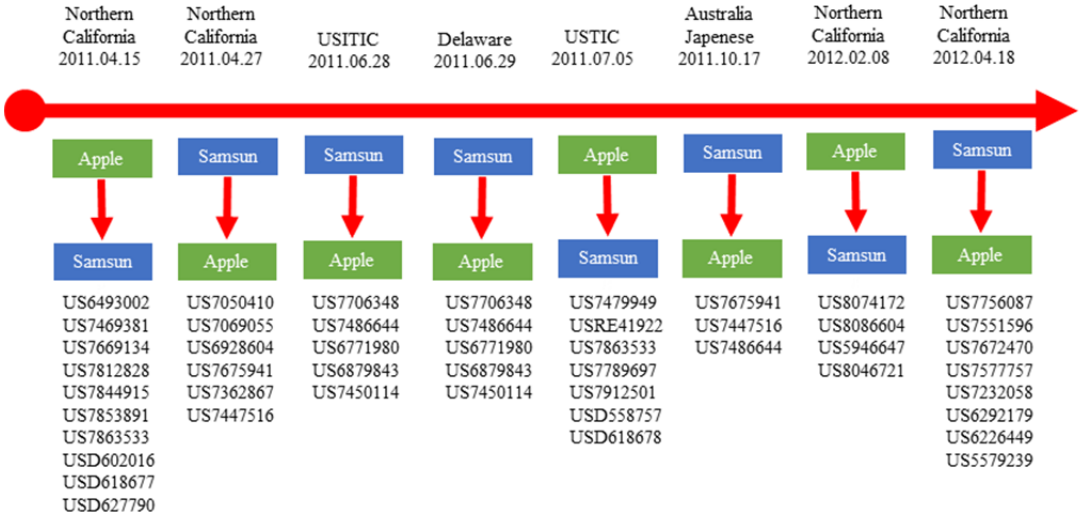
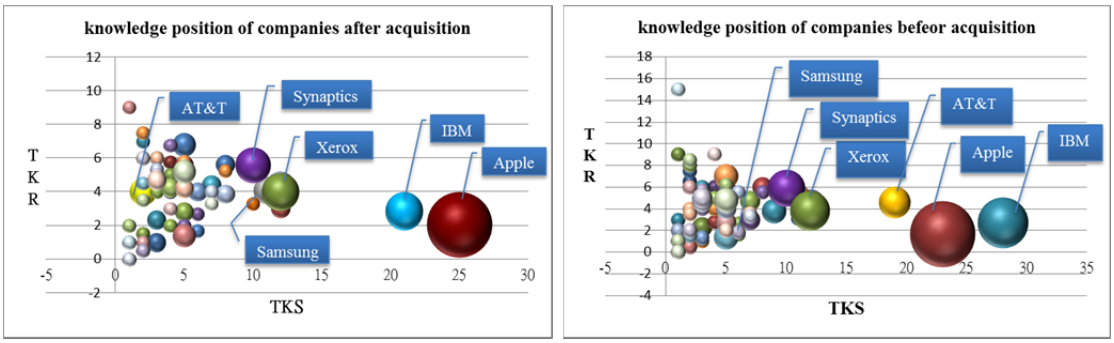


Fig. 4: Information regarding patent litigations between Apple and Samsung
Source: STPI

TABLE 1: PRODUCT PROPERTY AND EVALUATION INDEX
BEFORE/ AFTER PATENT TRANSFERENCE OF THE SIX MAIN COMPANIES

	company	Primary Products	TKS		TKR		patent counts	
			before	after	before	after	before	after
1	Apple	Mobile communication products, tablets, personal digital products	23	25	1.652	2.04	22	24
2	IBM	Information technology services and comprehensive hardware and software services for various industries	28	21	2.714	2.857	13	8
3	Xerox	Wireless control systems and relevant services for digitized photocopy products	12	12	3.833	4	8	8
4	Synaptics	Original equipment manufacturer for computer and laptop touchpads	10	10	5.9	5.6	7	7
5	Samsung	Mobile communication devices and consumer electronics	6	11	1.833	4.091	1	4
6	AT&T	Fixed-line telephone and mobile broadband services	19	2	4.579	4	5	4



*size of circle represents patent counts

Fig. 5: Knowledge Position of Companies before/after Patent Acquisition

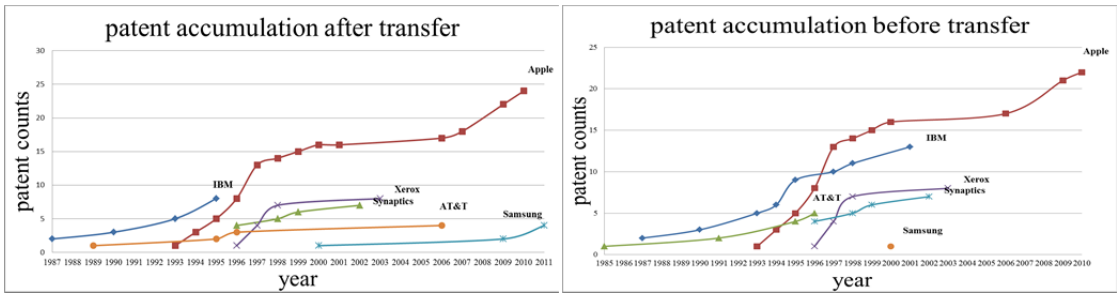


Fig. 6: The patent accumulation trends of the six major firms
before and after patent transactions

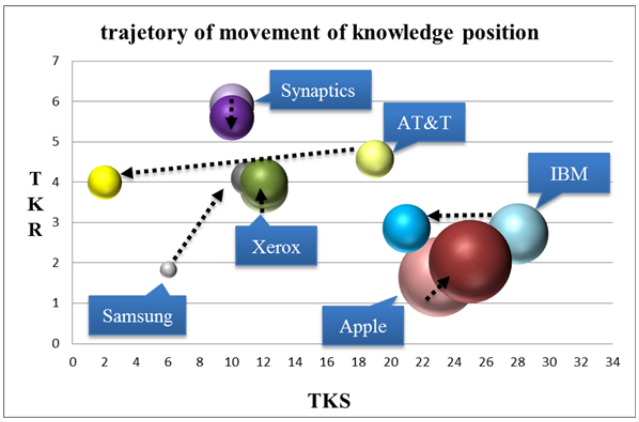


Fig. 7: Trajectory of Movement of Knowledge Position
before/after Patent Acquisition

The overlap of patent citation between companies shows the level of difference of technology. The movement of position in TKS-TKR diagram before/after patent transference could reveal the strategic intense of maintaining, transforming or withdrawing from a certain field of technology. The industrial organizations could decide to compete, cooperate or be coopetitive with others based on this analysis. We could locate the knowledge position in the technology structure in Fig. 5 and the trajectory of movement and change of position in Fig. 7. In the other word, any company's position moves, the other company moves, too. Especially, transference of the high complementary patents increases TKS a lot. TKR increases obviously if these patents are foundational technology also.

Apple has only a little change of TKS and TKR before/after patent transference in Fig. 7. This tiny change shows that Apple acquires supplementary technologies to maintain the leading position of the field in which Apple already leads. On the contrary, patents acquired by Samsung are complementary technologies which cause the increment of TKS and TKR being much bigger than Apple. Besides, the difference between Apple and Samsung in TKR is small (Apple =1.652；Samsung =1.833), but the gap between Apple and Samsung in TKS (Apple =23；Samsung =6) is huge before patent transference. That means these two companies have similarity technologies but much different technologic position. It makes sense that Samsung was Apple's loyal follower, parts supplier, ORM partner and dependent on Apple a lot before Apple brought patent suits against Samsung.

However, when Samsung started to develop its own brand on smartphone with Android system of Google, Apple found out Android system could cut out the market share of IOS and brought out patent suits against Samsung. On the other side, Samsung acquired patents to strengthen patent portfolios and technologic influence in order to enhance the bargain chips. The TKRs of Samsung, AT&T and Xerox changing little after patent transference show that they have similarity of technology. That means Samsung tries to strengthen the deployment in the field of wireless data transmission in order to increase the probability being complementary to Apple.

In addition, Xerox and Synaptics remain the same patent counts and their TKS and TKR change little before/after patent transference. That shows their position move little in the network. Their TKRs are bigger but TKSs are much smaller than Apple. They have steady position and role in this technology network and keep the probability of complementary to Apple.

IBM's TKS is bigger than Apple before transference but smaller than Apple after transference. The difference of TKS between these two companies is not large and IBM's TKR is a little higher than Apple. That shows both companies have equal technologic ability and IBM would like to create the opportunity to be complementary to Apple. Study infers that IBM tries to be a partner but competitor to every company based on its strategic role of integrated service provider. However, this message reveals that IBM could be a best partner but also a potential intimidator to Apple. It seems to be confirmed by that lots of Apple's products are related to the software of IBM.

The TKS of AT&T is giant before patent transference but falls much behind after transference. Its role and position are much less important. Based on Wired, Apple and AT&T is exclusive partner with each other. Apple's iPhones with 2G, 3G and 4G deploy the network of AT&T. However, they are not satisfied with each other due to the quality of data transmission. This study infers AT&T might withdraw from this market. This phenomenon is worth to observe.

B. Common Internal Knowledge, CIK and Common External Knowledge, CEK

CIK of Apple is greater than zero with each one of IBM, Xerox, Synaptics, Samsung and AT&T before/after patent transference in table 2. That represents Apple has technologies which are supplementary, dependency with each one of these five companies. The CIK with Samsung is largest, IBM is in the next place. Samsung shares a lot of common internal knowledge with Apple. On the other hand, IBM maintains almost 100% of complementary to Apple before/after patent transference. Both companies cooperate with each other in many fields of software and hardware. The CEK of AT&T with Apple is almost 100% before patent transference but becomes zero after transference. This result is related to that Apple is not happy with AT&T on transmission quality. To Samsung, CIK is big but CEK approaches zero with Apple. Samsung depends on Apple. However, it is not easy for Samsung to be complementary to Apple. Thus, both used to cooperate with each other before but compete against each other now. The cases of patent suit are still increasing and show no sign of stopping.

TABLE 2: CIK AND CEK BETWEEN APPLE AND OTHERS BEFORE/AFTER PATENT TRANSFERENCE

	Apple		IBM		Xerox		Synaptics		Samsung		AT&T	
transference	**B**	**A**	B	A	B	A	B	A	B	A	B	A
patent counts	**22**	**24**	13	8	8	8	7	7	1	4	5	4
patent overlap co-cited counts			4	3	1	1	1	2	5	5	1	1
CIK			0.182	0.125	0.045	0.042	0.045	0.083	0.227	0.208	0.045	0.042
CEK			1	1	0	0	0	0	0	0	1	0

Samsung is the important OEM partner and main supplier of Apple before April 15, 2011. Apple only needs to transfer a little of technology to Samsung for manufacture. Therefore, TKS of Samsung is small but TKR is as big as Apple. After April 15, 2011, because Samsung launches its own brand on smartphone and adopts Android OS of Google, Apple sues Samsung many cases for patent infringement. In order to respond patent suits, Samsung acquires external patents to enhance technologic power and bargain force. The patents acquired are not compatible to Apple. It is hard to create complementary value for both companies. Thus, TKR becomes bigger and CEK approaches zero.

Being the focal company, Samsung has one only patent but cites Apple five times. Its supplementary is as high as 500%, CIK is 5, before patent transference. However, its supplementary reduces to 125% due to four patents acquired after patent transference. Although Samsung acquires some external patents to reduce dependency on Apple and try to fight back, CIK is still too high showing that Samsung still over depends on Apple. Besides, because CEK of Samsung approaches zero, this research considers that the propose of acquiring patents for Samsung to increase bargaining power against Apple might fail.

In addition, CIK and CEK of Samsung with AT&T and Synaptics both approach zero before/after patent transference. This result shows Samsung has little overlap of technology and hardly cooperates with these two companies. However, presses report recently that Samsung's Galaxy S4 and Samsung Note 8 adopt Synaptics's chip to apply on the function of Air View and Samsung and AT&T work together to bring out Galaxy Note II, Galaxy Express, Galaxy Rugby Pro and Galaxy Tab 2 which all adopt Android and support 4G LTE. This study presumes this phenomenon might relate to the patent suits with Apple and Samsung might try to "de-Applize". We consider that CIK and CEK of Samsung between AT&T and Synaptics will increase as long as they cooperate with and depend on each other continuously.

VI. CONCLUSION AND SUGGESTIONS

A. Patent Detecting and Monitoring

Patent litigation has been an important strategy in the competetive technologic industy. Large major companies acquire external patents for rapid responding to market change in the dynamic competetive surrounding. In the other word, companies acquire patents as long as patent suits happen. Therefore, how do the companies proceed with supllementary and complementary innovation from their own technologies in order to respond to others' intentions[26]? That is the main quistion of this sdudy.

Competing in the same field of technology and market, companies have to detecte, monitor and respond to rival's intention no matter who is leader or follewer. This study surveys the patent acquirsition of the mobile telecommunication device manucaftures under dynamic competetion based on the concept of supllementary and complementary. Four patent indexes, TKS, TKR, CIK and CEK, evaluate the difference before/after patent acquirsition to avoid misjudaging and triggering improper reponse. Especially, the leader and follower become rivals fighting each other when the relationship of technologic supplementary OEM is finished.

B. Managerial Implication of Patent Index

This article adopts the concept of overlap of technologic knowledge to evaluate target patents for acquisition of two rivals to improve the quality of strategy making. Thus, this research adopts TKS and TKR as patent indexes which could detect the role, position and technologic similarity of companies and observe opponent's strategic intention of patent deployment. In a word, the technologic position formed by TKS and TKR is an index of coopetitive intention for companies sharing the same market.

Besides, technology similarity and knowledge similarity is the foundation of coopetitive relationship between two companies[11]. The change of common technologic knowledge could estimate the dependency between two rivals and help company to decide to cooperate with or compete against the opponent. Therefore, CIK and CEK are the evaluation indexes for strategy making in dynamic coopetition.

TABLE 3: CIK AND CEK BETWEEN SAMSUNG AND OTHERS BEFORE/AFTER PATENT TRANSFERENCE

	Samsung		Apple		IBM		Xerox		Synaptics		AT&T	
transference	**B**	**A**	B	A	B	A	B	A	B	A	B	A
patent counts	**1**	**4**	22	24	13	8	8	8	7	7	5	4
patent overlap co-cited counts			5	5	0	1	0	1	0	0	0	0
CIK			5	1.25	0	0.25	0	0.25	0	0	0	0
CEK			0	0	0	0	0	0.143	0	0	0	0

C. Role of Leader and Follower

Companies purchase either supplementary or complementary asset[5, 17]. The strategy of patent acquisition is different for the leader and follower. The leader, Apple, has higher position in the technology ego-network and its TKS increasing after acquisition shows Apple tries to protect and raises the position of its unique technology. TKR going high as well as TKS shows Apple's technologies that are applied and adopted by followers and diffuse in the market immediately. We consider that Apple with high patent cited counts should search reliable technologic supply partner. To the incumbent leader, sustaining the technology on hand and integrating resource are its main concern to construct its unique technologic capability to avoid pressure from market. Facing followers' intimidation, the leader should positively expand the partnership with more than one manufactures based on its unique technologic capability.

Besides, this article classifies followers into two types, high TKS/low TKR oriented and low TKR/high TKS oriented. First, high TKS/low TKR oriented ones with more internal citations but less external citations have unique technologies but not mainstream technologies. Secondly, low TKR/high TKS oriented ones have less unique technologic capability and develop their technologies depending on external support and integration. For the long-term competitiveness, they should acquire crucial patents to catch up with leader rapidly.

Samsung's TKR is bigger than Apple before/after patent acquisition that shows Samsung has a lot of activities cooperating with others. The high priority of the kind of follower like Samsung is not building high position of unique technology but preying on market and increasing manufacture power for OEM. However, Samsung reveals ambition on communication technology with its raising TKS after acquisition. Furthermore, it is a crucial coopetitive problem of Apple depending too deep on Samsung's manufacture. Apple's choice of partner of its supply chain will influence the balance of smartphone industry.

D. Future Research

This research compares only Apple and Samsung from April 15, 2011 to April 18, 2012. Recent data should be collected for future research because of quick change in the smartphone industry. In addition, future research could analyze all companies in the smartphone industry or the objects of different industries. Build up the reference for scholars and practical operation according patent citation network.

REFERENCES

[1] Kanzatec LLC. (march 5). *http://www.kanzatec.com/about.htm*.

[2] M. J. Chen, "Competitor Analysis and Inter-Firm Rivalry: Towards a Theoretical Integration,".*Academy of Management Review,* vol. 21(1), pp. 100-134, 1996.

[3] H. Anderson, Havila, V., Andersen, P. & Halinen, A., "Position and role - conceptualizing dynamics in business networks.," *Scandinavian Journal of Management,* vol. 14(3), pp. 167-186, 1998.

[4] K. G. Smith, Ferrier W.J. & Ndofor H., *Competitive dynamics research: Critique and future directions, Handbook of strategic management*. London: Blackwell Publishers, 2001.

[5] D. J. Teece, "Profiting from technological innovation: Implications for integration, collaboration, licensing and public policy," *Research Policy,* vol. 15, pp. 285-305, 1986.

[6] L. C. Freeman, "Centrality in social networks: Conceptual clarification," *Social Networks,* vol. 1, pp. 215-239, 1979.

[7] T. E. Stuart, J. M. Podolny, "Local Search and the Evolution of Technological Capabilities," *Strategic Management Journal,* vol. 17, pp. 21-38, 1996.

[8] J. M. Podolny, T. E. Stuart, M.T. Hannan, "Networks, Knowledge, and Niches: Competition in the Worldwide Semiconductor Industry, 1984-1991," *American Journal of Sociology,* vol. 102, pp. 659-689, 1996.

[9] B. Yoon, Park, Y., "A text-mining-based patent network: analytical tool for high-technology trend," *Journal of High Technology Management Research,* vol. 15, pp. 37-50, 2004.

[10] S. Breschi, Lissoni, F., *Handbook of Quantitative Science and Technology Research, the Use of Publication and Patent Statistics in Studies of S&T Systems*. Springer Netherlands, 2005.

[11] M. A. H. Marianna makri, Peter J. Lane, "Complementary Technologies, Knowledge Relatedness, and Invention Outcomes in high Technology mergers and acquisitions," *Strategic Management Journal, Strat. Mgmt. J,* vol. 31, pp. 602-628, 2010.

[12] B. Wellman, Berkowitz, S. D., *Social Structures: A Network Approach*. New York: Cambridge University Press, 1988.

[13] J. Scott, *Social network analysis: A handbook*, 2nd ed. London: Sage Publications, 2000.

[14] T. E. Stuart, "Network position and Propensities to Collaborate: An investigation of strategic alliance formation in a high-technology industry," *Administrative Science Quarterly,* vol. 43, pp. 668-698, 1998.

[15] J. Coleman, "Social capital in the creation of human capital," *American Journal of Sociology,* vol. 94, pp. 95-121, 1988.

[16] S. a. F. Wasserman, K., *Social Network Analysis: Methods and Applications*. UK: *Cambridge University Press*, 1994.

[17] D. J. Teece, "Reflections on "profiting from innovation"," *Research Policy,* vol. 35, pp. 1131-1146, 2006.

[18] C. Cobuild, *English Dictionary*. London: HarperCollins Publishers, 1997.

[19] A. Rindfleisch, Moorman, C., "The Acquisition and Utilization of Information in New Product Alliances: A Strength of Ties Perspective," *Journal of Marketing,* vol. 65, pp. 1-18, 2001, April.

[20] G. B. Richardson, "The Organization of Industry," *The Economic Journal,* vol. 82, 1972.

[21] S. J. Chen, Su, F. P., Lai, K. K., Yang, M. T., Chang, P. C., "The Patent Information, Strategic Patent Deployment Thinking, and Technology Strategies of Small and Medium-Sized Enterprises," in *PICMET*, San Jose, California, 2013.

[22] A. Knudsen, "Islamism in the Diaspora: Palestinian Refugees in Lebanon," *Journal of Refugee Studies,* vol. 18, pp. 216-234 2005.

[23] G. Ahuja, "Collaboration Networks, Structural Holes, and Innovation: A Longitudinal Study," *Administrative Science Quarterly,* vol. 45, pp. 425-455, 2000.

[24] B. V. Cassiman, R., "In search of complementarity in innovation strategy: internal R&D, cooperation in R&D and external technology acquisition," *Management Science,* vol. 52, pp. 68-82, 2006.

[25] STPI. (2012, March 5). *http://cdnet.stpi.narl.org.tw/techroom/pclass/2011/pclass_11_A116.htm*.

[26] C. D. Mowery, Oxley , J. E., Silverman, B. S., "Technological Overlap and Interfirm Cooperation: Implications for the Resource-based View of the Firm," *Research Policy,* vol. 27, pp. 507-523, 1998.

APPENDIX

Algorithm of TKS, TKR, CIK and CEK

Step 1: construct the affiliate network of firms and patents

$$M = [\alpha_{kr}]_{g\times h},\ \alpha_{kr} = \begin{cases} 1 \text{ if } P_k \text{ affiliatd to } A_r & k = 1,2,\dots,g \\ 0 \text{ else} & r = 1,2,\dots,h \end{cases}\ g \geq h \qquad (1)$$

k : k^{th} patent, i : i^{th} firm, g : patent counts, h : firm counts

Step 2: Technological Knowledge Status, TKS

$$[TKS_{ii}]_{h\times h} = M^T M,\quad TKS_{ii} = \sum_{k=1}^{g} \alpha_{ik}\alpha_{ki} \quad i = 1,2,\dots,h \qquad (2)$$

α_{ik} : k^{th} patent affiliated to i^{th} firm, g: patent counts, h: firm counts

Step 3: Technological Knowledge Reliability, TKR

$$\left[TKR_{ij}\right]_{h\times h} = M^T M,\quad TKR_{ij} = \sum_{k=1}^{g} \alpha_{ik}\alpha_{kj} \quad \begin{matrix} i=1,2,\dots,h \\ j=1,2,\dots,h \end{matrix}\ i \neq j \qquad (3)$$

TKR_{ij} : overlap of knowledge between i^{th} firm and j^{th} firm

$$TKR_{ii} = \frac{\sum_{j=1}^{h} TKR_{ij}}{TKS_{ii}} \quad i = 1,2,\dots,h \text{ and } j = 1,2,\dots,h,\ i \neq j \qquad (4)$$

TKR_{ii} : overlap of knowledge of i^{th} firm with others

Step 4: Common Internal Knowledge, CIK

$$CIK_{ij} = \frac{\sum \alpha_{ik_o}\alpha_{jk_e}}{\sum \alpha_{ik_o}} \quad \begin{matrix} i=1,2,\dots,h \text{ and } j=1,2,\dots,h,\ i\neq j \\ o=1,2,\dots,n \text{ and } e=1,2,\dots,n,\ n<g \end{matrix} \qquad (5)$$

Approval date of patent k_o of i^{th} firm must be earlier than Approval date of patent k_e of j^{th} firm

Step 5: Common External Knowledge, CEK

$$CEK_{ij} = \frac{TKR_{ij} - \sum \alpha_{ik_o}\alpha_{jk_e}}{TKS_{ii} - \sum \alpha_{ik_o}} \quad \begin{matrix} i=1,2,\dots,h \text{ and } j=1,2,\dots,h,\ i\neq j \\ o=1,2,\dots,n \text{ and } e=1,2,\dots,n,\ n<g \end{matrix} \qquad (6)$$